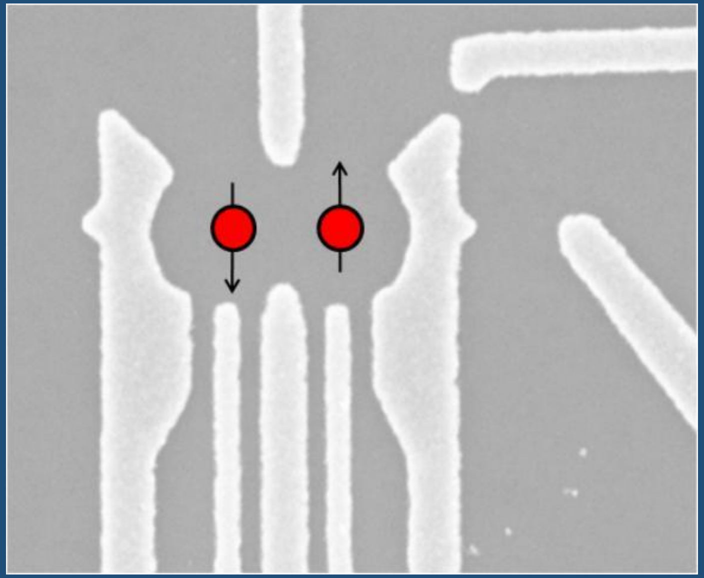
Theoretical Exploration of Resonant Tunneling in Quantum Dot Systems: A New Approach to Performing Joint Measurements



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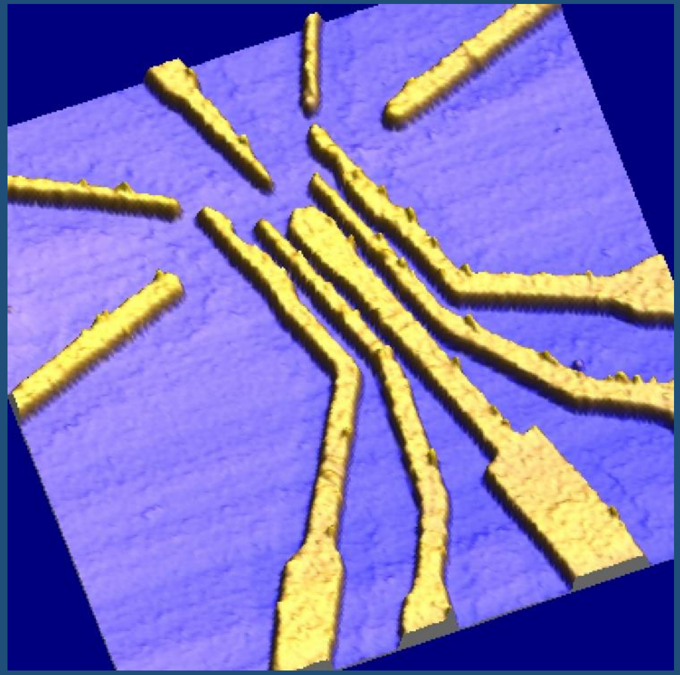

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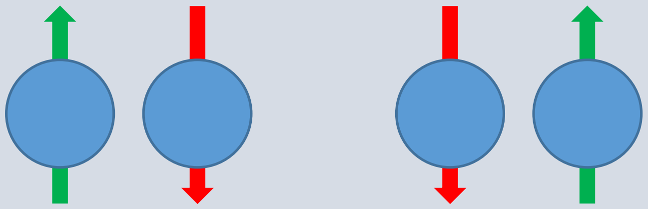
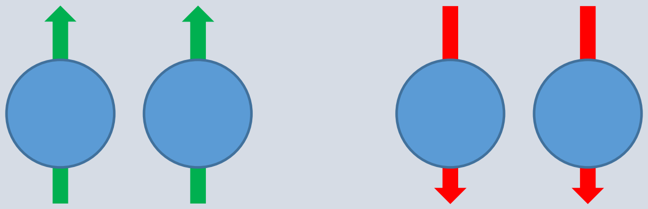
Introduction

Odd parity



Even parity



- Joint measurement on multiple qubits gives information about the system without revealing the individual state of each qubit

Application:

- Quantum entanglement
- Basis of fault-tolerant error correction procedures

Theoretical conditions must be taken into account before this procedure can be implemented in experiment

Quantum Tunneling

By enforcing continuity conditions, the outgoing amplitudes can be obtained in terms of the incoming amplitudes. This operation can be generalized to an effective transfer matrix.

$$\begin{pmatrix} C_{n1} \\ C_{n2} \end{pmatrix} = M_{n-1}M_{n-2}...M_2M_1M_0 \begin{pmatrix} C_{01} \\ C_{02} \end{pmatrix}$$

$$M_m = \begin{pmatrix} \frac{k_{m+1}+k_m}{2k_{m+1}} e^{-i(k_{m+1}-k_m)d_m} & \frac{k_{m+1}-k_m}{2k_{m+1}} e^{-i(k_{m+1}+k_m)d_m} \\ \frac{k_{m+1}-k_m}{2k_{m+1}} e^{i(k_{m+1}+k_m)d_m} & \frac{k_{m+1}+k_m}{2k_{m+1}} e^{i(k_{m+1}-k_m)d_m} \end{pmatrix}$$

$$T = \frac{|F|^2}{|A|^2} = \frac{|M|^2}{|M_{22}|^2}$$

$$R = \frac{|B|^2}{|A|^2} = \frac{|M_{21}|^2}{|M_{22}|^2}$$

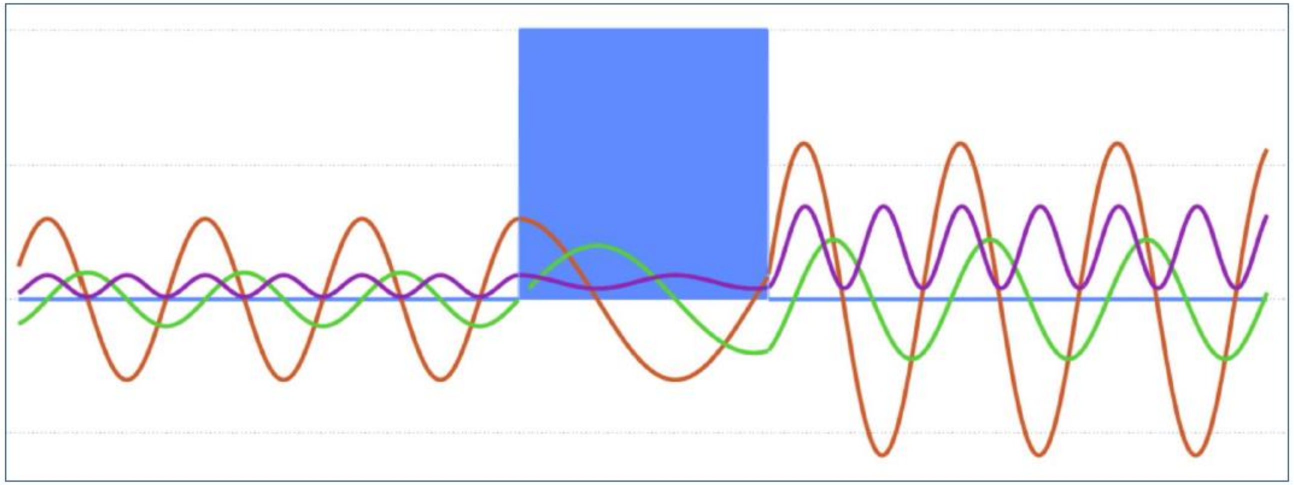


Figure 1. Scattering across a rectangular barrier. The real part is orange, the imaginary part is green, and the square of the wavefunction is purple.

Time-independent Schrodinger equation:

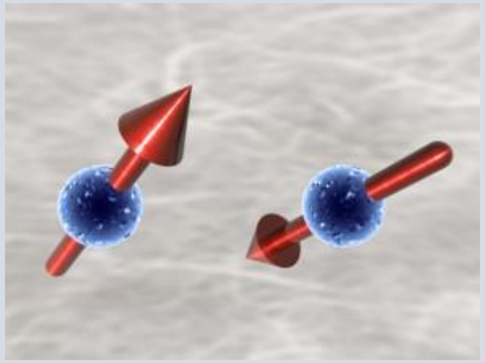
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi(x) = E\psi(x)$$

Wave function:

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$k = \sqrt{\frac{2m(E-V)}{\hbar^2}}$$

Quantum Dot



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- Artificial GaAs/AlGaAs heterostructure that holds electrons whose spin state forms a **physical qubit**
- Joint measurement: **Transport electrons** in a nearby conductance channel
- **Electrostatic coupling** between qubits and channel creates a **scattering potential** that depends on the **spin states** of the trapped electrons

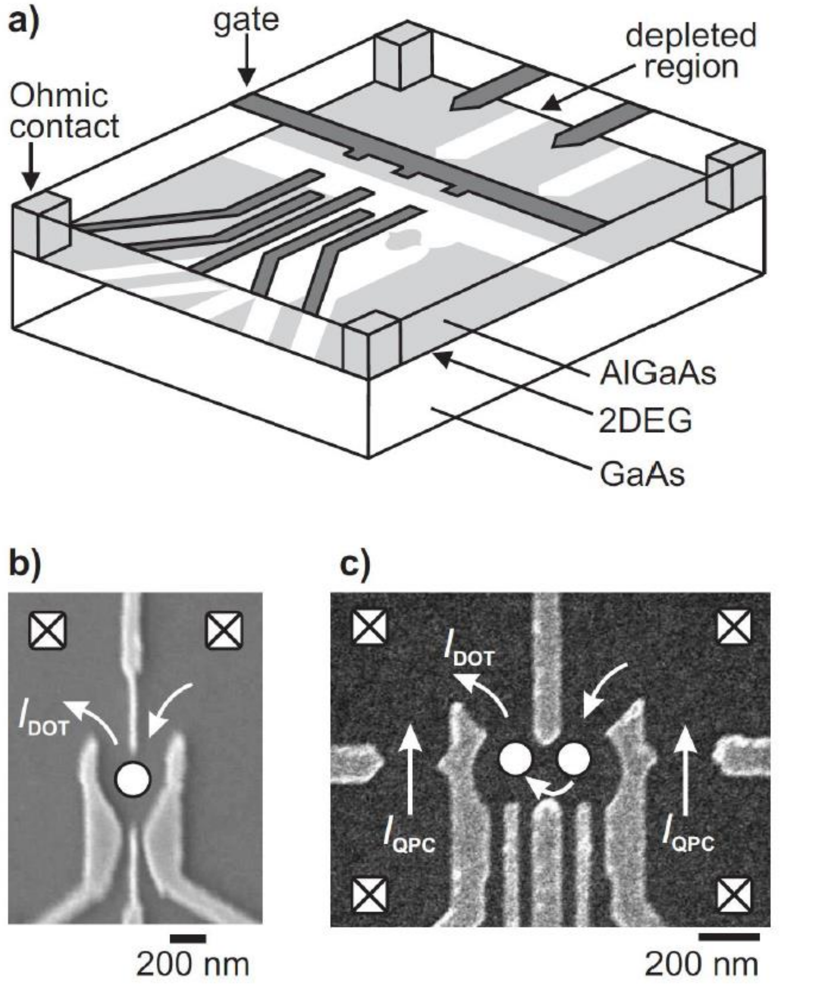
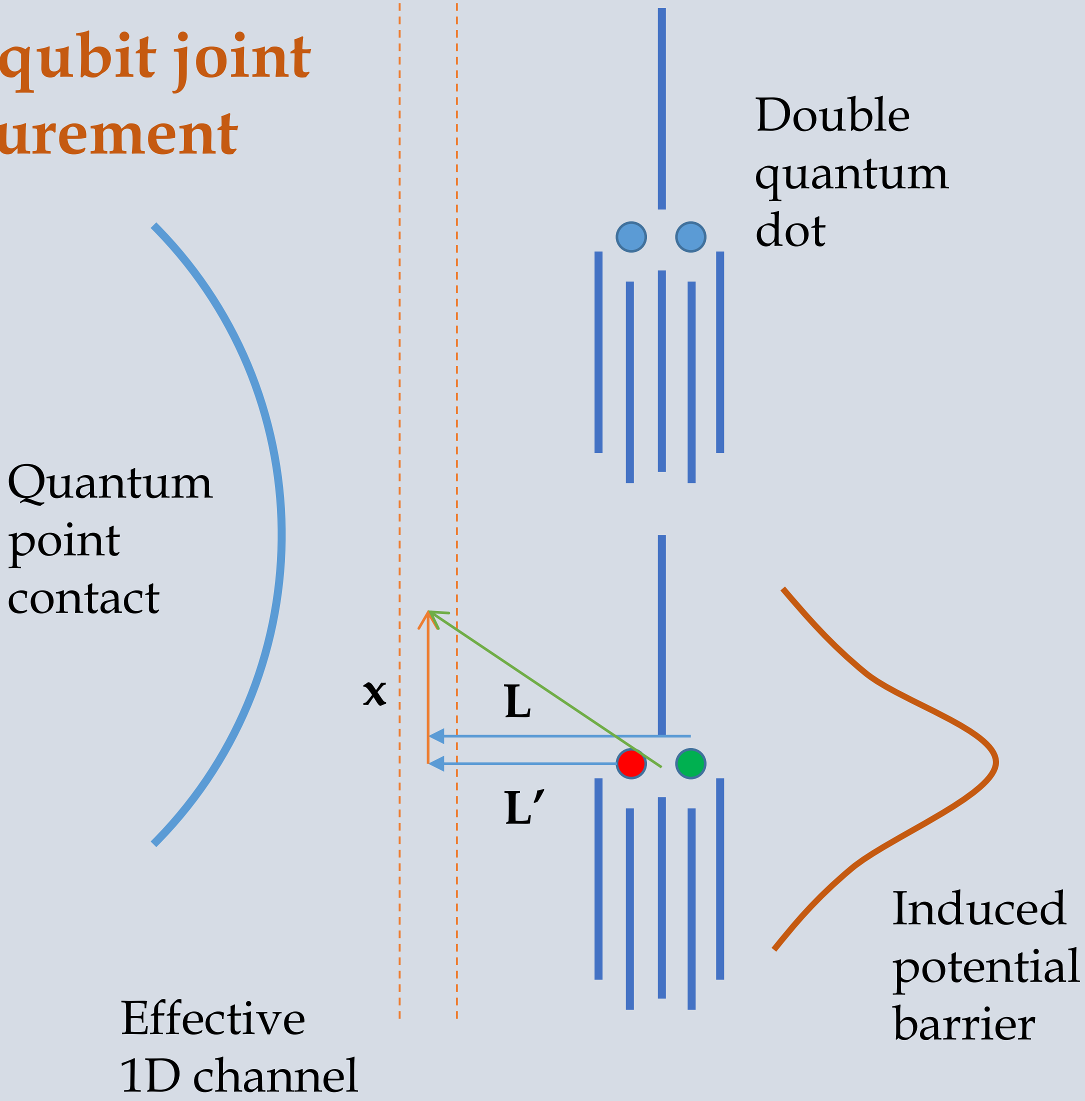


Figure 2. Quantum dot. (a) Voltages applied by gate electrodes (dark gray) deplete regions in the 2DEG. (b) White dots indicate location of the electrons. (c) Current passing through the side will be measured (Hanson et al., 2007).

Barrier Potential

Two-qubit joint measurement



For a conductance channel in a GaAs/AlGaAs heterostructure, the ideal electric potential induced by a coupled qubit in the channel is:

$$V_1(x) = \frac{\delta e}{e}\frac{e^2}{4\pi\epsilon}\left(\frac{1}{\sqrt{x^2+l'^2}} - \frac{1}{\sqrt{x^2+l^2}}\right)$$

where the spins are placed in dots l=200 nm away from the channel and turning up the spin results in a fraction δe/e of the charge density tunneling into a dot l′=100 nm away.

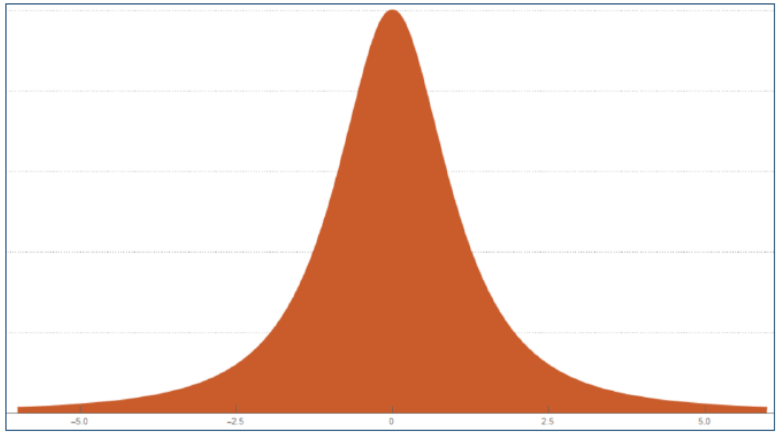


Figure 3. Shape of induced potential barrier.

Current Measurement

Thermal-broadening function

$$F_T(E) = \frac{1}{4k_BT} \operatorname{sech}^2\left(\frac{E - E_0}{2k_BT}\right)$$

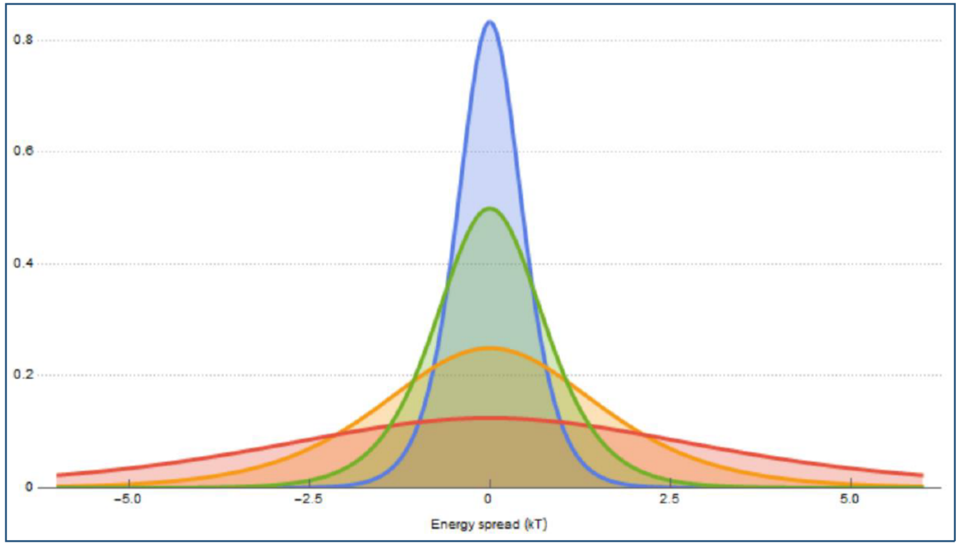


Figure 4. In a realistic model, incident electrons have a distribution in energy described by the thermal-broadening function.

In experiment, the current passing through the conductance channel will be measured.

$$I = \frac{e}{h}\int T(E)F_T(E)\,dE$$

$$I \approx \frac{e\Delta E}{4k_BTh}\sum_{i=0}^{n-1} T(E_1 - E_0 + i\Delta E) \operatorname{sech}^2 \frac{E_1 + i\Delta E}{2k_BT}$$

Shot noise must also be taken into account which is due to the discrete nature of electric charge:

$$I_{shot} \sim \sqrt{I\frac{e}{2\Delta t}}$$

The difference in currents must be larger than the shot noise to distinguish between the two states.

Resonant Tunneling

The experimental constraints on distance sensitivity limit precision to 5-10 nm. Similarly, the spread in the electrons' energy is constrained by the lowest temperatures achievable in dilution refrigerators with typical base temperatures around 20 mK. For purposes of distinguishability, the state-of-the-art devices can detect deviations in the current by **1%** which will be set as a critical value.

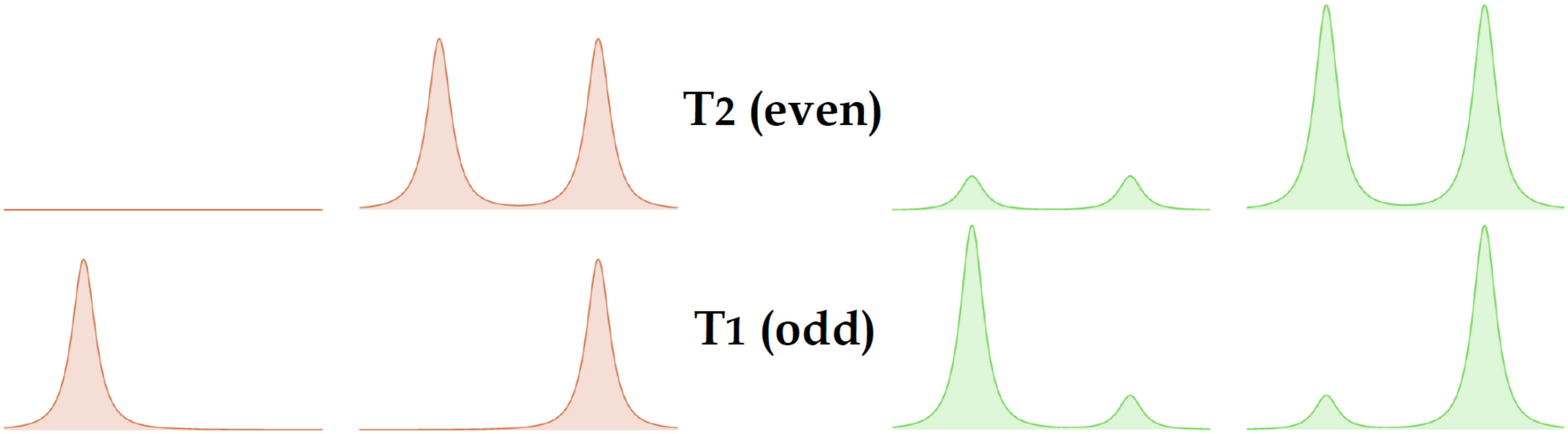
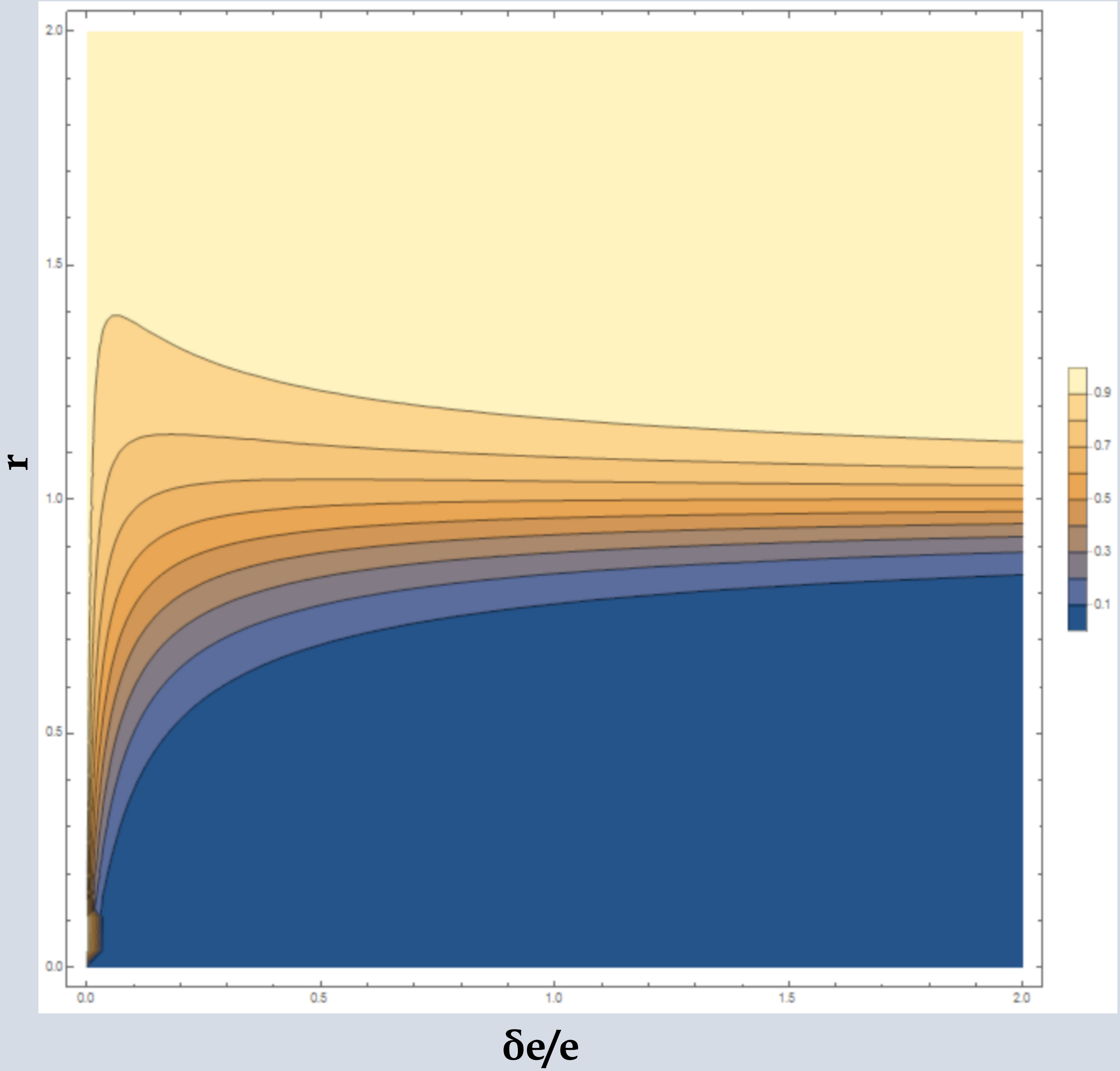
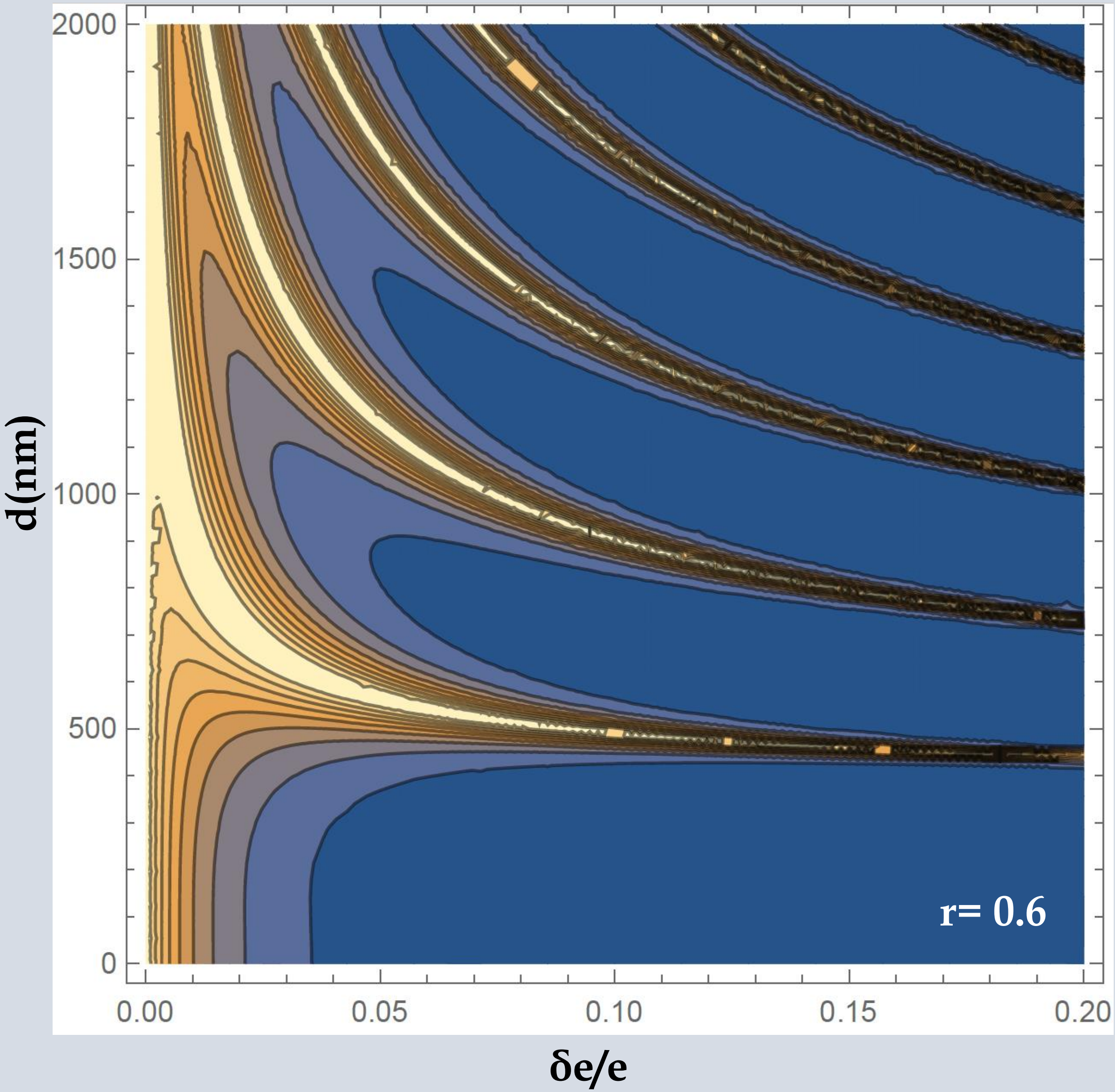


Figure 5. Transmission across potential barriers. Standard scattering in orange, with added background term (+b*V(x)) in green.

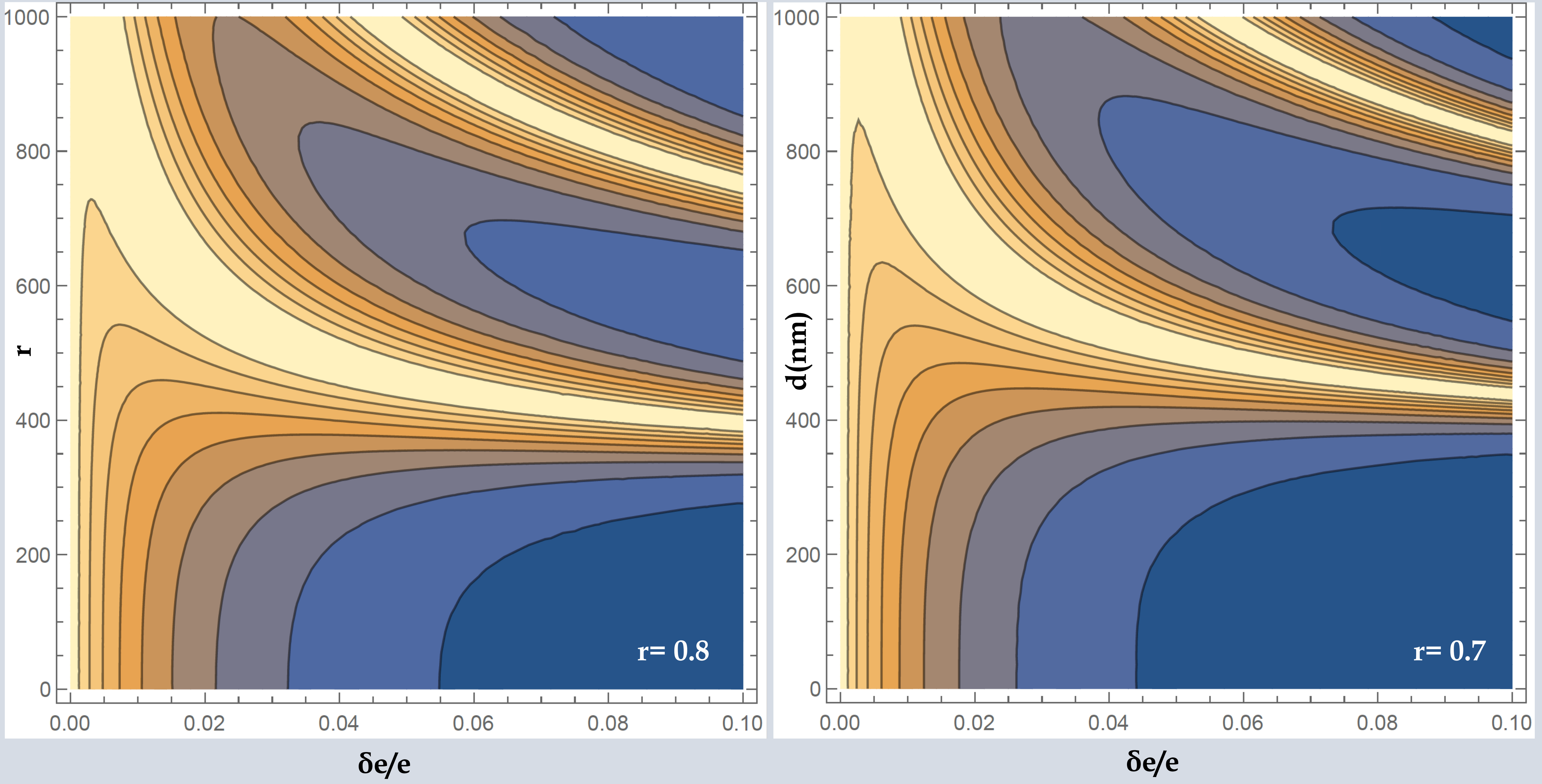
Results

r is defined as the ratio of energy to barrier height.





Two barrier transmission contour

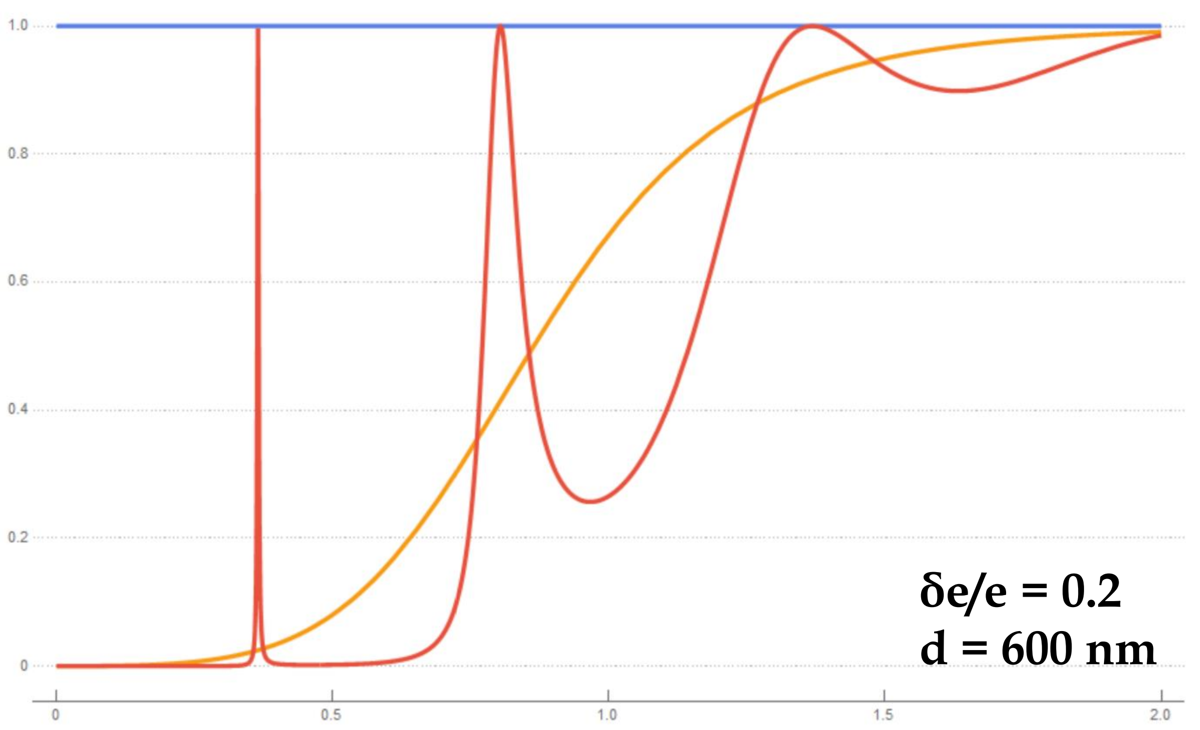


Transmission vs. Energy Plots

2-qubit case (without background term)

3-qubit case (without background term)

T



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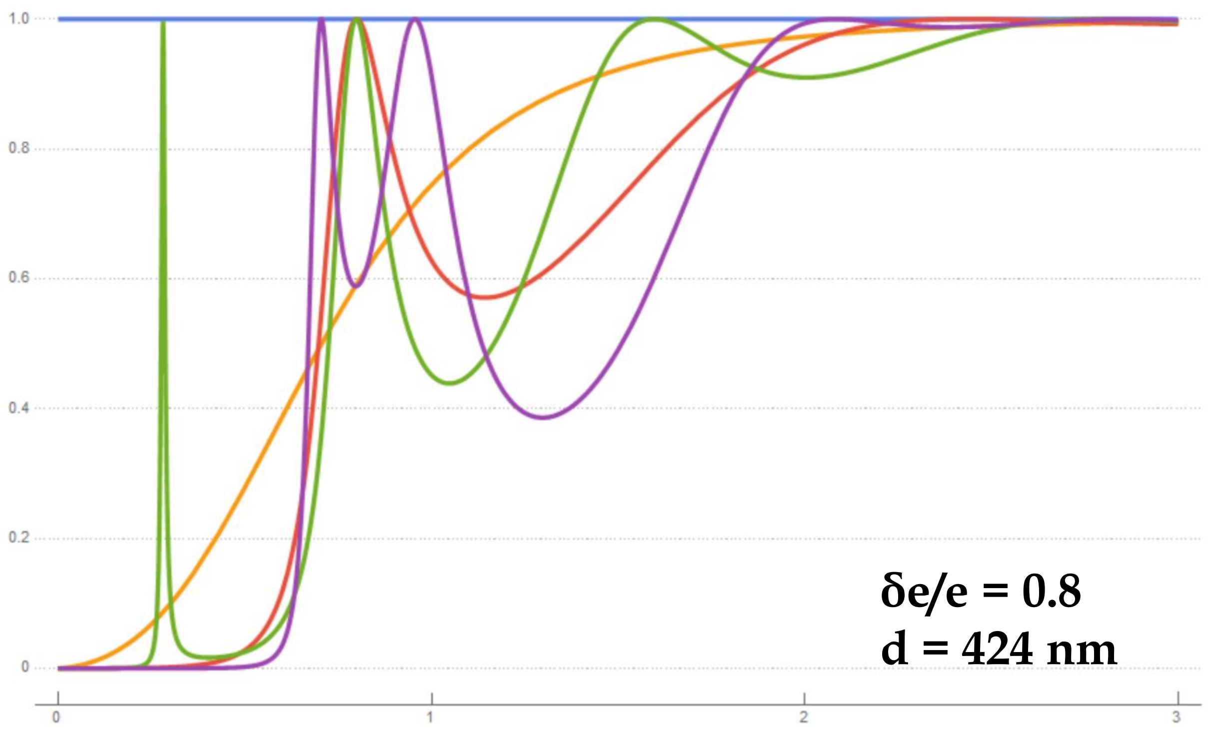

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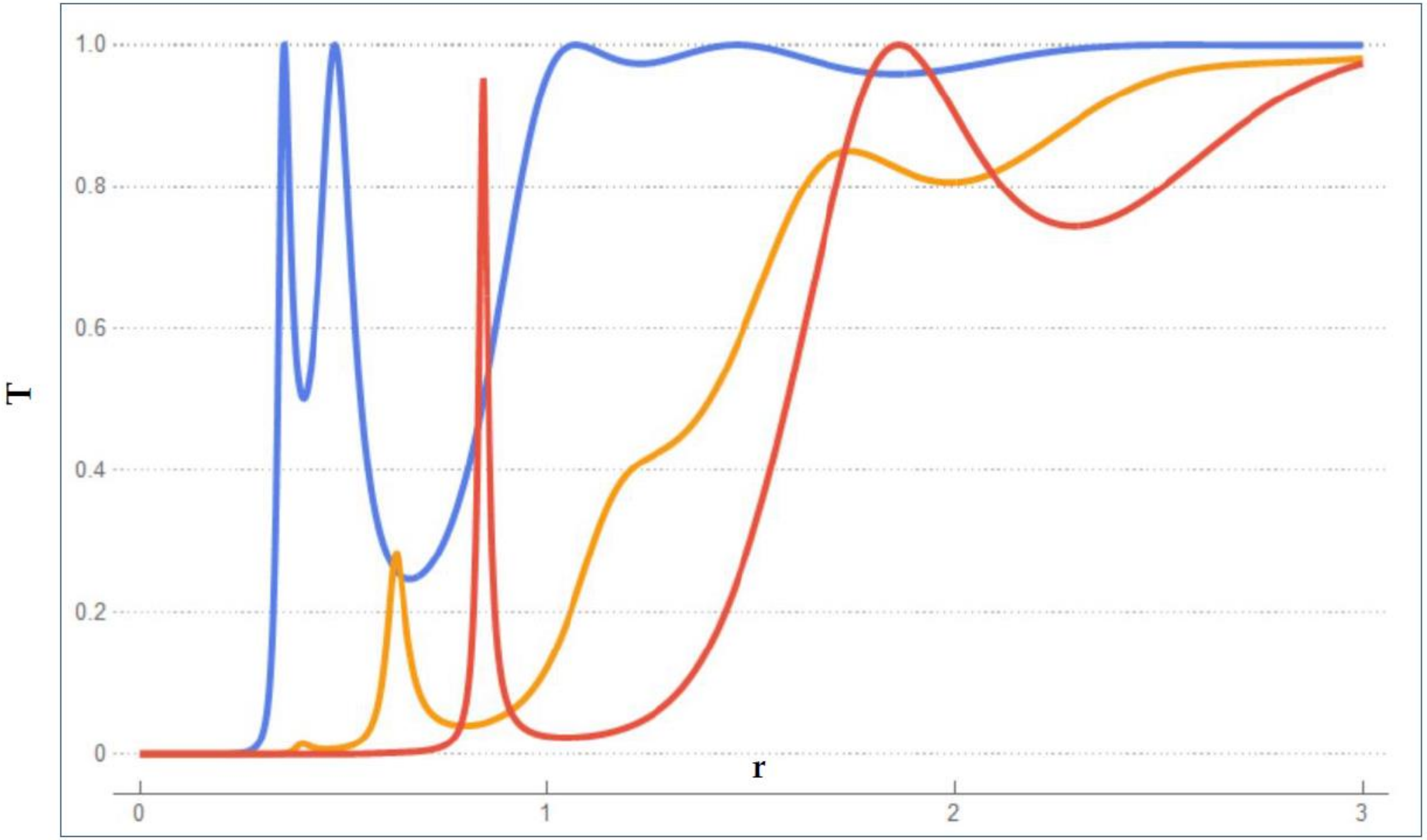


Figure 7. Transmission plot for two-barrier potential with background term that satisfies realistic constraints.

Sample solution for two-qubit joint measurement

Set-up: **T = 25 mK, b = 0.6, d = 500 nm, δe/e = 0.1, r = 1.86 (E = 102 ueV)**

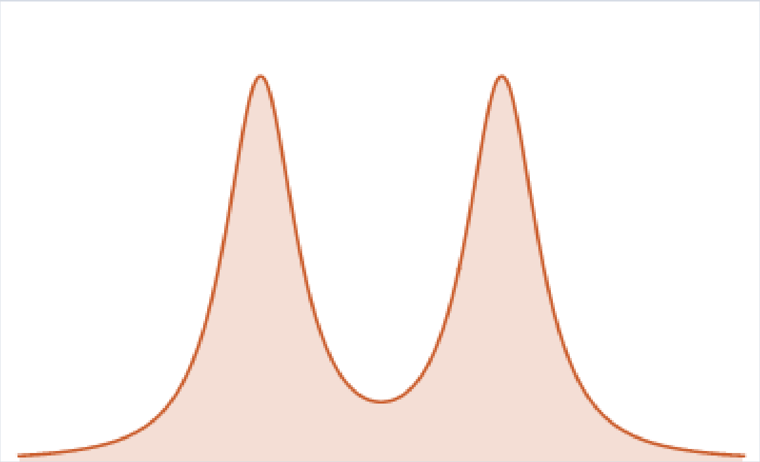
I_0 = 2.047 nA, I_2 = 2.055 nA, I_1 = 1.760 nA

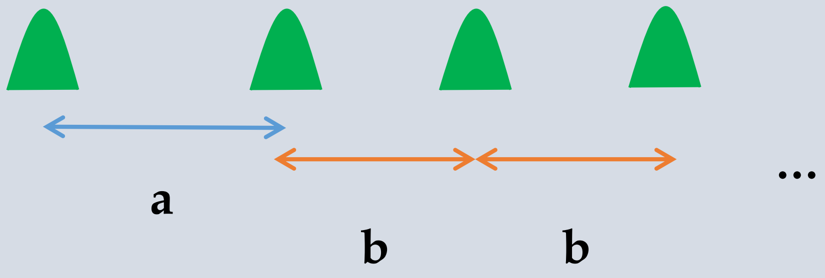
I_0-I_1 = 0.287 nA, I_{shot} = 0.029 nA

Discussion

- **Compromise** between distance and energy **sensitivity** is observed
- Unlike rectangular barriers, these resonant distances display a general linear pattern **a+nb** which leads to a **discrete set** of solutions in the case of more than two qubits
- For **a=mb**, the sensitivity in distance would be decreased below the **sub-10 nm** range

Overlap between potential barriers leads to loss of resonance



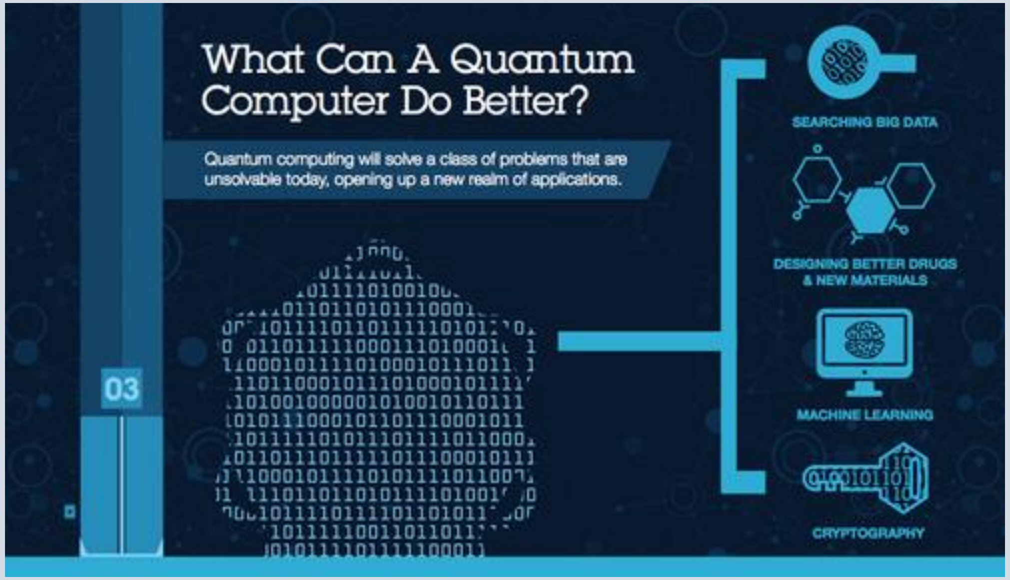


Conclusion

- Important step forward to **experimental implementation**
- For both the **two-qubit** and three-qubit joint measurement, the main challenge is dealing with the **spread** in the **electrons' energy**
- Transmission restriction on allowed solutions is absent in the two-qubit case where an added **background term** satisfies **experimental constraints** considered
- Conclusions drawn here lay the **foundation** for further paths of exploration to maintain parameters within **realistic bounds**

Future Direction

- Expand upon preliminary result to 2D and **3D detailed device modelling**
- Calculate the sensitivity of the joint measurement in the presence of additional components of **noise** and **decoherence**
- Developing efficient quantum entanglement and error correction procedures is essential to **building quantum computers**



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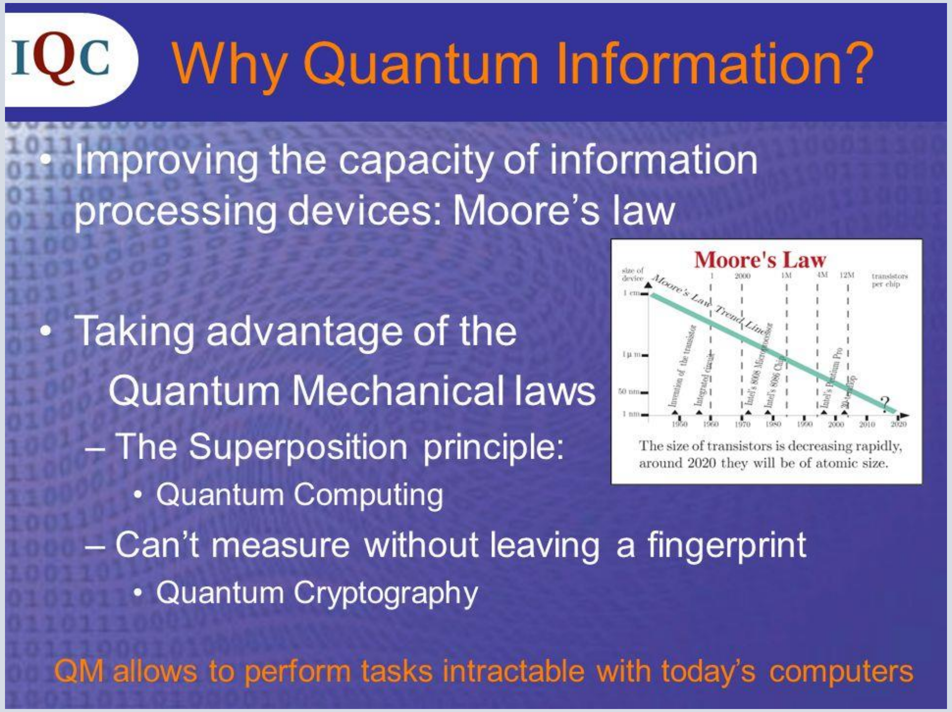

uwaterloo.ca/institute-for-quantum-computing

Selected References

Hanson et al. (2007). Spins in few-electron quantum dots. *Reviews of Modern Physics*, *79*(4).

Reilly et al. (2007). Fast single-charge sensing with a rf quantum point contact. *Applied Physics Letters*, *91*(16).

Vandersypen et al. (2004). Real-time detection of single-electron tunneling using a quantum point contact. *Applied Physics Letters*, *85*(19).